**Energy**

**Example**

Let’s compare this prediction to the actual value for silver. The molar mass of silver is approximately mmol. = 108g/mol. Then according to our model we shouuld have,



which compares pretty well to the actual value of cp = 0.234 kJ/kg˚C.

**Example:**

How fast is an O2 molecule moving, supposing that the temperature is T = 30˚C?

The mass of this molecule would be 16g per mole approximately, which works out to be:



Boltzman’s constant is k = 1.38×10-23 JK-1. So plugging these into the formula, and noting that its translational kinetic energy has 3 degrees of freedom,



So v = 686m/s ≈ 1544 mph!

**Example:**

Estimate how much kinetic energy is contained in a room filled with such a gas?

Each molecule has a total kinetic energy of KE = (5/2)kT. Room temperature is around 300K. All we need to know now is how many molecules are in a room full of gas. We can use the equation of state to figure this out. Estimate the dimensions of a room to be 7m×5m×3m = 105m3. The pressure of the gas is 1atm = 1.013×105 Pa. So we have,



So the total kinetic energy would be:



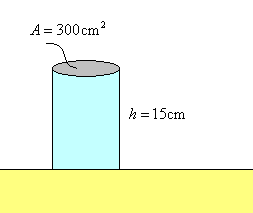
This is quite a bit of energy.

**Example**

do one comparing energy required to heat room up from something to something, compared to energy of moving car, flywheel, falling water. And explain how can extract work from latter, and desire to extract work from former as well.

**Example:**

Suppose we have 2 mol of a gas of Helium atoms (mHe atom = 6.70×10-27kg). Suppose it occupies a cylinder with cross sectional area and height as shown. If the temperature of the gas is T = 300K, what is the total energy of the gas?



The mechanical energy is:



Note that there is no mechanical kinetic energy because the cylinder is not moving, itself. Also, to calculate the gravitational potential energy we use the height of the center of mass of the cylinder, i.e., the middle of it. Now for the internal energy. Note that there is no chemical potential energy because in a gas, all of the atoms are disconnected from eachother, freemly roaming around the container. So the energy is, using the results from the previous lecture:



So the total energy would be:



As you can see, an object’s internal energy is usually much larger than its mechanical energy.

13. A balloon is filled with He (which is a monatomic gas) at room temperature T = 25˚C. What is the speed of the Helium atoms in the balloon? Use the fact that the He atom has a mass of about 6.68×10-27 kg.

The kinetic energy of such an atom is given by:



**Example**

How much internal energy must be extracted to freeze 65kg of water at an initial temperature T = 30˚C?

We can use the heat capacities to figure this out. So first we figure out how much energy to take out to lower the temperature of the water to 0˚C, and then how much energy to take out to convert all of this water to ice. So first, to lower the T…



**Example**

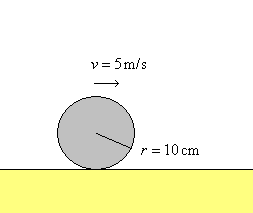
By approximately how much does the internal energy of 1m3 of water increase if you compress it to a volume of 0.95m3?

According to the formula, the increase in internal (potential) energy would be:



**Example**

Suppose we have an iron ball of radius 10cm, rolling to the right at a speed of 5m/s. Let its temperature be T = 300K. What is its mechanical energy? What is its approximate internal energy? What is its approximate total energy ?



First we need the mass of the ‘iron’ ball, and the number of moles of ‘iron’ contained. The density of iron is: ρ = 7847kg/m3, and its molar mass is: mmol = 55.85g/mol. So the mass of iron is:



and the number of moles is:



So, the mechanical energy is:



As far as the internal energy, this is approximately,



So the total energy is approximately,



Again, the internal energy is the largest component by far.

**Example**

Suppose you heat the iron ball up to 350˚C. What will be its new internal energy?

When you heat up the iron it will expand, and so its internal potential energy will increase since the ‘springs’ lengthen. Also its temperature is increasing and so its internal kinetic energy and potential energy will increase as well since the spped and amplitude of the oscillations of the atoms will increase. The volume expansion is given by:



And so the change in internal potential energy from the volume change and temperature change will be:



So we can see that most of the energy change comes from the increased speed and amplitude of the oscillations of the atoms, rather than from the expanssion of the springs themselves.

**Energy Transfer + 1st law**

**Example**

How much heat must be extracted to freeze 65kg of water at an initial temperature T = 30˚C?

We can use the heat capacities to figure this out. So first we figure out how much energy to take out to lower the temperature of the water to 0˚C, and then how much energy to take out to convert all of this water to ice. So first, to lower the T…



Now to freeze it:



So the total heat/energy which needs to be extracted to freeze the water is:



14. A block of ice (with dimensions 10cm × 10cm × 10cm) at temperature T = 0˚C is sitting outside in the shade. The temperature outside is T = 30˚C. Assuming it is a perfect emitter and absorber of radiation (ε = 1), how long will it take to completely melt? Assume heating occurrs via radiation only. What is the change in entropy of the block of ice in problem 15?

The block will radiate heat according to:



and absorb energy according to:



So rate of incoming energy is:



Now, the amount of heat it must absorb to melt is:



So the amount of time it will take to melt is given by:





**Example**

Approximately how long would it take a sphere of water (mwater = 65kg) at initial temperature T = 30˚C to freeze if it were released in outer space? Take the emissivity to be ε = 1.

First the amount of heat the water must loose in order to freeze was worked out in a the previous problem. We found Q = 2.97×107J.

Next, let’s figure the mechanism of this heat transfer and then how long it will take to freeze. There is no way for conduction or convection to take place in space. So the only heat loss would come from radiation. To calculate this heat loss we need the surface area of the sphere. We can get this from the volume,



So the radius is:



and the surface area is:



Therefore the heat per unit time radiated out into space is:

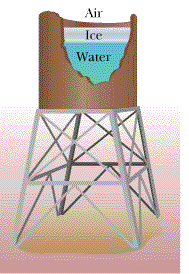


So to see how long it will take for this to occur we write,



This is about 22 hours – nearly a day – a lot longer than you’d think.

5. A tank of water has been outdoors in cold weather, and a slab of ice 5 cm thick has formed on its surface (see the figure). The air above the ice is at -15°C. Calculate the rate of formation of ice (in centimeters per hour) on the ice slab. Take the thermal conductivity of ice to be 0.0040 cal/s·cm·C°, its density to be 0.92 g/cm3, and its latent heat of fusion to be 333 kJ/kg. Assume no energy transfer through the tank walls or bottom.



The instantaneous rate at which heat is leaving the tank is:



From 1st law applied to water below ice slab we have:



Filling our dQ/dt expression into dx/dt we get:



**Question 2**. The Sun’s intensity at Moon’s surface is about I = 1360 W/m2. Estimate the temperature of the Moon’s surface, assuming that its emissivity is about ε = 1. You can take surrounding space to be at a temperature of 0K. Assume that only half of the moon’s total surface area radiates heat.



**Example**

Suppose you live in a house whose walls are 20cm thick and filled with air. And suppose that the total surface area of the walls is A = 180m2. If the temperature inside is 25˚C and outside is 0˚C, what is the rate of heat loss through the walls via conduction?

Heat loss would occur via conduction, and radiation (and perhaps convection a bit), but we’ll focus on the conduction. Heat would be conducted from the inside of the house across the walls to the outside. The rate of heat loss would be:



And this is also the amount of heat you would have to pump into the house to keep it at a constant temperature.

**Question 2**. The Sun’s intensity at Moon’s surface is about I = 1360 W/m2. Estimate the temperature of the Moon’s surface, assuming that its emissivity is about ε = 1. You can take surrounding space to be at a temperature of 0K. Assume that only half of the moon’s total surface area radiates heat.



**Example**

Suppose 8cm diameter and 14cm tall cup of coffee at 90C is insulated by a roughly cylindrical cup whose bottom, sides, and lid have thickness 7mm, and thermal conductivity k = 0.05W/m∙K. The air temperature is 25C. What is the initial rate of heat loss? Approximating as constant, how long until it cools to 85C?





In problem above, suppose that you were to take the lid off instead. Then what would have been the initial rate of heat loss? Assume convenction as main mechanism of heat loss through the time, and that h = 20 W/m2K.



And and how long would it take to cool to 85 in this case?



**Example**

Estimate how long would it take a cup of coffee at temperature T = 85˚C to cool to room temperature T = 25˚C? Assume the coffee cup to have a diameter of 5cm, and a height of 5cm as well.

The coffee will cool according to the three mechanisms – conduction, convection, and radiation. First let’s figure out how much energy must be lost by the coffee (water) in order to cool to 25˚C. The mass of water in the cup is



And the specific heat of water is 4186 J/kg·K. So we need an energy loss of:



The rate of heat loss due to convection is approximately,



The cup of coffee will also lose heat due to radiation. The rate of this heat loss will be:



The cup would also lose heat due to conduction, but this effect would be small in comparison to the other too. So the net rate of heat loss will be:



and so the approximate time it will take for the coffee to cool down is:



**Example**

Suppose an asteroid, composed mainly of silicon, heats up as it plows through the atmosphere. If it hits the ground with an initial temperature of T = 1000cC, and completely vaporizes, estimate its speed upon impact.

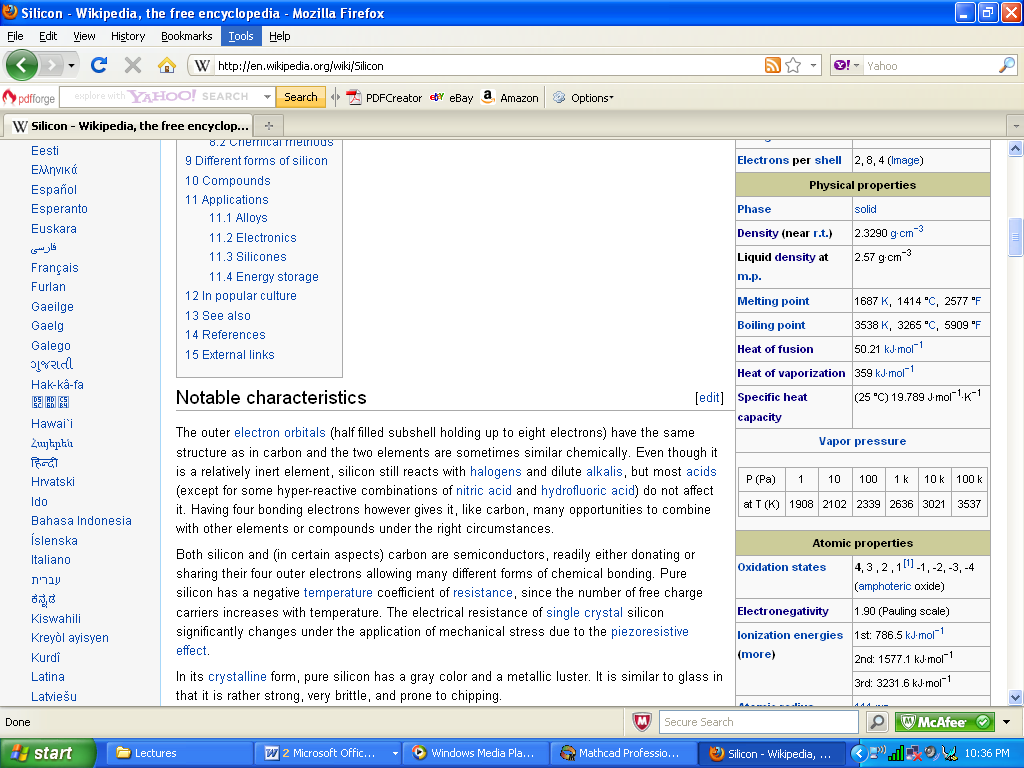
Now apply the 1st law. We have,



We’ll take as our system the asteroid. We’ll assume for simplicity that most of the asteroid’s Emech. = (1/2)mv2 gets converted to thermal energy, Eint. and relatively little goes into Wn.c. on the Earth, or gets transferred to the Earth as heat. So then we have,



Now ΔEmech. = (1/2)m02 – (1/2)mv2 since it goes from some velocity v, to velocity 0 upon impact. And ΔEint. will look like this. First its temperature will increase to its melting point, then it will melt, then its temperature will increase to its vaporization point, and then it will vaporize. Looking up the properties of Silicon on Wikipedia…



So,



where I’ve used the molar mass conversion factor 28.1g/mol to convert the molar specific and latent heats to just plain mass specific and latent heats. Continuing…

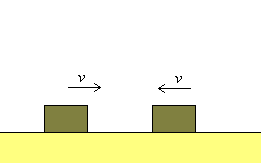


Note that it is the heat of vaporization that contributes most to the change in internal energy. Anyway, continuing with our 1st law equation,



**Example**

Suppose you slide two pennies towards each other at a speed of 1.5m/s, on a frictionless surface. As a result of the collision they both come to rest (probably not going to happen but we’ll suppose so for the sake of argument). Assuming the pennies absorb all the energy from the collision, and don’t deform, what is the temperature increase of the pennies as a result? Suppose the initial temperature is room temperature T = 25C˚.



So what is happening is that all of the kinetic energy of the pennies will be transferred into internal energy of the pennies. To calculate the temperature increase, we will use the 1st law of course, and apply to our system which we’ll take to be the two pennies together. So the first law says,



The reasoning is this. The only external forces acting on the pennies are gravity and the normal force, and they do not do any work on the pennies. So Wnc. = 0. Energy will transfer out of our system in a bunch of different ways. As the pennies heat up as a result of the collision, they will radiate, convect heat into the air and ground. Also, during the collision sound waves will be emitted which also carry energy. So this total energy transfer Q, is not equal to 0. Nonetheless we will assume it is quite small and so Q ≈ 0. This isn’t a bad approximation. There is a change in Emech. of course because the pennies slow down to rest, and therefore there must be a ΔEint. (due to temperature increase) as well.

Continuing we have,



where we used the mass of a penny, m = 2.5g (look up on Wikipedia). To get the change in temperature associated with this change in internal energy we’ll use the heat capacity appropriate to zinc (pennies are 97.5% Zn). This is (again, according to Wikipedia):

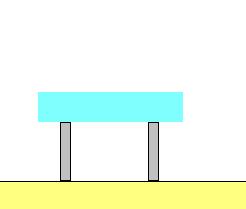


So we’ll have,



**Example**

Estimate how fast you need to punch a slab of ice (we’ll say it measures 10cm×10cm×50cm) in order to split it in two.



So when we karate chop the block of ice, the energy transfer will look like this. Take our system to be the block of ice + hand. Then we have,



The reasoning is this: No external work is done on our system so Wnc. = 0. Some heat may be transferred out of our system via radiation/convection. Also sound waves from the impact will carry energy out of our system. But we will assume these to be small so Q ≈ 0. The change in energy of system will look like this. Our hand will slow down to v = 0 upon impact and so it will have a change in KEmech.. Assuming no temperature increase in our hand, there will be no change in KEint. and assuming no deformation of our hand, like broken bones, etc., there will be no change in PEint. of our hand. As for the block, assuming its temperature doesn’t increase, there will be no change in its KEint., but there will be a change in its PEint. since it will deform (shear to be particular) under the force of our hand. This will change the chemical potential energy in the block.

So continuing, the mass of our arm is approximately 5kg maybe. What is the change in PEint. of the ice? Let’s use our approximation from the previous file.



where we assume that there is only a change in potential energy due to the shear – there is no change in volume of the ice, and there is no temperature increase (probably not strictly true). Now S ≈ B/3 = 3×109 Pa, and for Δx = 0.005L0 (because fractures occur for stresses of about 0.005 for ice). And L0 = 25cm, the length from one end of the ice to your hand. Filling in these numbers we get,

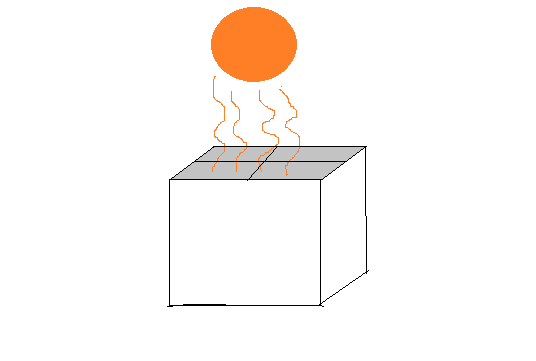


Continuing with the first law we have,



which seems like a reasonable estimate. Of course, a certain technique is involved in making sure all of your hand/arms kinetic energy goes into the block’s elastic potential energy.

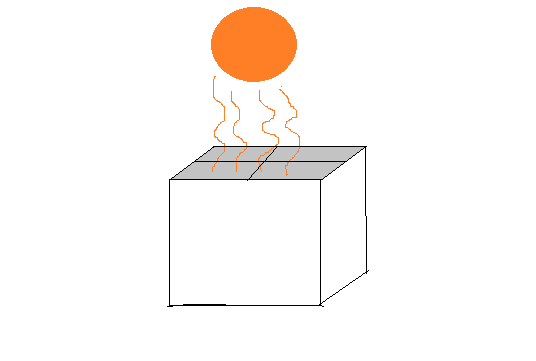
**Question 3**. Suppose a cubic shed has a window for a roof, and is under direct sunlight. Let the sunlight intensity be I = 1200 W/m2 and 20% be reflected from the window. The walls of the shed are made of wood, with thermal conductivity k = 0.17 W/m∙°K and thickness 5cm. Finally the ambient air temperature is 20°C. What will be the equilibrium temperature of the shed? (note *thermal* radiation doesn’t play a role here since the surface temperature of the outside of the shed will be the same as the ambient temperature)





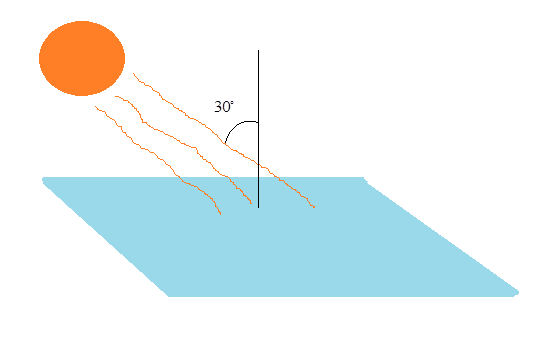
So that means inside tempeature should be T = 20 + 70.6 = 91C. Ouch.

**Question 3**. Suppose a cubic shed has a window for a roof, and is under direct sunlight. Let the sunlight intensity be I = 1200 W/m2 and 10% be reflected from the window. The walls of the shed are made of wood, with thermal conductivity k = 0.12 W/m∙°K and thickness 5cm. The thermal conductivity of the window is about 0.95 W/m∙K, and has thickness 1cm. Finally the ambient air temperature is 20°C. What will be the equilibrium temperature of the shed? (note *thermal* radiation doesn’t play a role here since the surface temperature of the outside of the shed will be the same as the ambient temperature)





3. Suppose we have a pool of water. Sunlight, of intensity I = 1200 W/m2 shines on it at an average angle of 30°, 15% of which is reflected. The ambient temperature is on average T = 300K, and we’ll presume εthermal = 1. So what do we expect that equilibrium temperature of the lake to be?





**Problem**

In situation above, assume that main heat loss term is evaporation. Let A = 100m2. If temperature of lake remains constant at about T = 300K, same as ambient temperature, what would be the rate of evaporation?



**Problem 4.** Model yourself as a cylinder h = 1.8m tall and with circumference C = 60cm. If your skin temperature is T = 35°C, and the ambient temperature is T = 25°C, at what rate do you lose heat to the environment? You make assume εth = 1. And don’t forget the top and bottom of the cylinder!

We have:



**example**

Estimate how much water evaporates in world oceans. Assume Earth is just water.



**Question 1**.

Sunlight is incident on asphalt at an angle of 20° with respect to the vertical. The sunlight’s intensity is 1250 W/m2, and 8% of it is reflected from the surface. The surrounding air temperature is about 25°C. What temperature will the asphalt equilibrate to, assuming the ashphalt loses heat mostly through convection? You may take the convection coefficient to be 15 W/m2°K.



**Entropy**

**Example**

Ice has a specific heat of about 2kJ/kgK, a latent heat of fusion Lf 333kJ/kg. Water has a specific heat of about 4.18kJ/kgK, and a latent heat of vaporization Lv = 2256 kJ/kg. Water vapor has a specific heat of about 2kJ/kgK also. What is the change in entropy of a 55g cube of ice as it is heated from -20C to 120C?

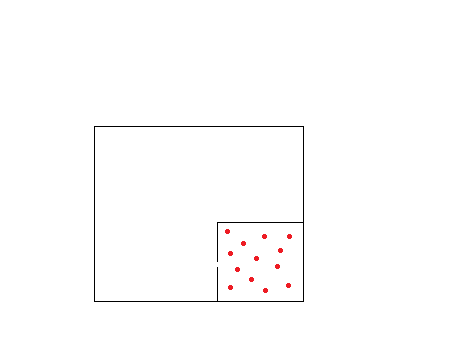


**Example**

Most substances’ heat capacities are temperature-dependent. Suppose one has heat capacity c = 0.75T2 kJ/kg∙K. What is the change in energy of 1.3kg of this substance as its temperature is raised from 0 to 90K. What is its change in entropy?



**Question 3**. Suppose you have 0.77 mol of gas kept in a container of volume V = 0.04m3, pressure p = 5atm, and temperature T = 400K. Then you open the container and let it expand into a larger insulated container of volume V = 0.20m3 (this volume includes the original container). (a) What will be the change in energy of the gas? (b) What will be its change in entropy?



Change in energy is 0. Change in entropy is:



5. Suppose you have a gas kept in a container of volume V = 0.02m3, pressure p = 3atm, and temperature T = 300K. Then you open the container and let it expand into a 10m×10m×3m room. What will be its change in entropy?



**Question 2**.

2.5 kg of a mysterious substance is heated from -75°C to 120°C. It’s specific heat capacity is temperature dependent, given by c(T) = (1 + T2) kJ/kg°K. What is its change in entropy?



**Question 8**. A triatomic gas held at temperature 90K, volume 0.004m3, and pressure 0.3atm, slowly leaks out of its container and fills the room, which has a volume of 62m3, and is maintained at room temperature T = 298K. If the gas equilibrates to room temperature as well, what will be its change in entropy?



**2nd law**

8. An insulated Thermos contains 0.3kg of hot coffee at 90°C. You put in a 20g ice cube at its melting point to cool the coffee. By how many degrees (in Celsius) has your coffee cooled once the ice has melted and equilibrium is reached? Treat the coffee as though it were pure water and neglect energy exchanges with the environment.

Using the 1st law of thermodynamics,



Working out this problem from a slightly different perspective than I usually do, adhering more closely to the book…there is no net internal energy change in the entire system. And moreover no work is done on any of the parts and so we have:



Breaking up the heat transfers into the heat transfer for each entity we have:



1. An insulated Thermos contains 0.4kg of hot coffee at 90°C. You put in a 25g ice cube at its melting point to cool the coffee. When the system equilibrates, what will have been the total change in entropy?

First we have to find the new equilibrium temperature. This follows from the 1st law:

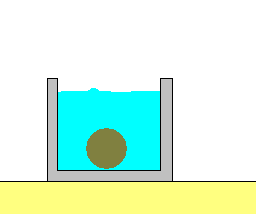


And then the total change in entropy would be:



**Example**

A copper block whose mass, mc, is 75g is heated in a laboratory oven to a temperature T = 312C. The block is then dropped into a glass beaker containing a mass mw = 220g of water. The heat capacity Cb of the beaker is 45cal/K. The initial temperature Ti of the water and beaker is 12C. What is the final temperature Tf of the mixture when thermal equilibrium is reached?



What will happen is that heat from the copper block flows out and into the water, and beaker. Thus the copper block’s temperature will drop, and the water, and beaker’s temperature will rise. This will continue until all three temperatures are the same.

Mathematically, it works like this. Let’s take our ‘object’, i.e., our system, to be the beaker + water + copper block (sphere). Using the 1st law we can say:



where we recognize first that no work is being done by our system so Wnc. = 0. Secondly, heat will be conducted across the walls of the beaker, convected and radiated into the air as well. So the heat transferred out of our system, Q isn’t equal to 0. Nonetheless we will assume that we’re insulating the beaker really well and so Q ≈ 0 even still. Solving for T we have:



so there we are.

**Question 1**. A 50g ice cube at -20°C is placed in a 1.5kg Aluminum bucket whose initial temperature is 30°C. What will be the final equilibrium temperature?



**Question 2**. What will be the overall change in entropy for the situation above?



9. A block of ice (m = 1kg) at temperature T = 0°C sits in room full of air, mair = 20kg, at T = 25°C. If the heat capacity of air is cv = 0.717kJ/kg, and the heat of fusion of ice is Lf = 333kJ/kg, and the heat capacity of water is 4.18kJ/kg, what will the final equilibrium temperature of the room be?

Using the 1st law of thermodynamics,



2. An insulated Thermos contains 0.35kg of hot coffee at 90°C. You put in a 15g ice cube at its melting point to cool the coffee. At what temperature will the objects equilibrate?

First must find equilibrium temperature. Using 1st law,



3. What is the change in entropy of the system above?

In Kelvin, this temperature would be: T = 356K. So then the change in entropy will be:



**Question 1**. A 50g ice cube at -20°C is placed in a 1.5kg Aluminum bucket whose initial temperature is 30°C. What will be the final equilibrium temperature?



**Question 2**. What will be the overall change in entropy for the situation above?



**Example**

Why can’t a block at temperature T, convert an amount Q of its thermal energy into kinetic energy? How exactly would this be a violation of the 2nd law of thermodynamics?

The 2nd law says that ΔSsys. ≥ 0 for a closed system. The block would constitute a closed system because no energy would be going into or out of it from the outside. Now its change in entropy would be:



which is < 0. And this would be a violation of the 2nd law.

**Example**

Show two substances will come to same temperature. And calculate what it is.



and,



Solving for T1f in terms of T2f, we’d have:



and so now differentiating w/r to T2f and setting equal to 0 we have:



and for T1i we have:

**Question 3.**

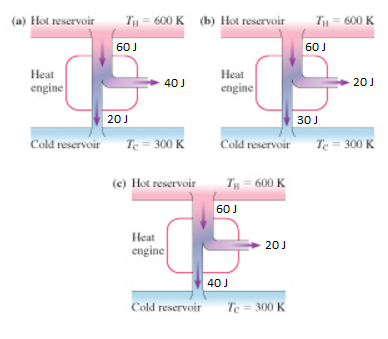
A block of ice at 0°C is added to a well-insulated 0.30kg aluminum calorimeter cup that holds 0.20kg of water at 50°C. The water and aluminum cup are in thermal equilibrium, and the specific heat of aluminum is 0.910 kJ/kg°K. If the final equilibrium temperature of the mixture is 35°C, what was the mass of the ice block?



**Question 1.**  You’ve bought a 16 oz cup of coffee (mcoffee = 0.45kg). It’s temperature is T = 95°C. You want to bring its temperature down to 80°C by dumping some -10°C ice into it. How much ice should you use? Note the specific heat of ice is about 2kJ/kg∙°K. And you can treat the coffee as water.



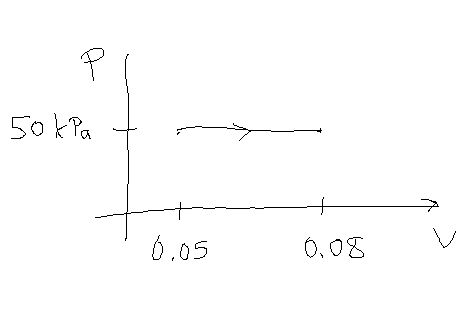
**Question 9**. State which, if any, of the first and second laws of thermodynamics, the following engine cycles violate. Please justify ☺.



1. Since 60 = 40 + 20, the first law is satisfied, and since -60/600 + 20/300 = -0.033 < 0, the second law is violated.
2. Since 60 ≠ 20 + 30, the first law is violated. But since -60/600 +30/300 = 0 ≥ 0, the second law is barely satisfied.
3. Since 60 = 20 + 40, the first law I satisfied. And since -60/600 + 40/300 = 0.033 ≥ 0 the second law is satisfied.

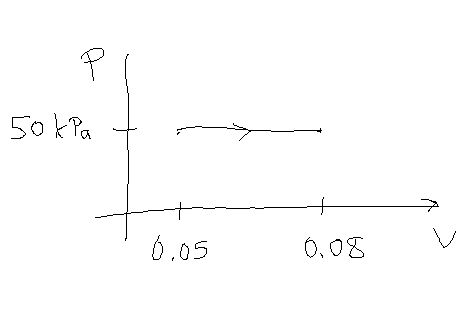
**Processes**

4. N2 gas at pressure p = 50kPa is isobarically expanded from a volume of V1 = 0.05m3 to V2 = 0.08m3. Draw this process in a pV diagram. And then determine the change in internal energy of the gas.



ΔE = (f/2)NkΔT = (f/2)NkΔT. And from the equation of state: pV = NkT, whereby ΔT = pΔV/Nk. Filling this in, we have: ΔE = (f/2)Nk∙pΔV/Nk = (f/2)pΔV = (5/2)(50∙1000)(0.03) = 3.75 kJ.

4. N2 gas at pressure p = 50kPa is isobarically expanded from a volume of V1 = 0.05m3 to V2 = 0.08m3. Draw this process in a pV diagram. And then determine the change in internal energy of the gas.



ΔE = (f/2)NkΔT = (f/2)NkΔT. And from the equation of state: pV = NkT, whereby ΔT = pΔV/Nk. Filling this in, we have: ΔE = (f/2)Nk∙pΔV/Nk = (f/2)pΔV = (5/2)(50∙1000)(0.03) = 3.75 kJ.

**Example**

Suppose universe is 13 billion light years in radius, and temperature is about T = 2K, and pressure is p = 10-11 Pa. Then estimate the number of particles in the universe.

**Example**

What would have been the temperature of the universe, moments after the big bang, when it had a radius of 1m?

We’ll take f to be 3 just cause most particles in space are by themselves. And so then:



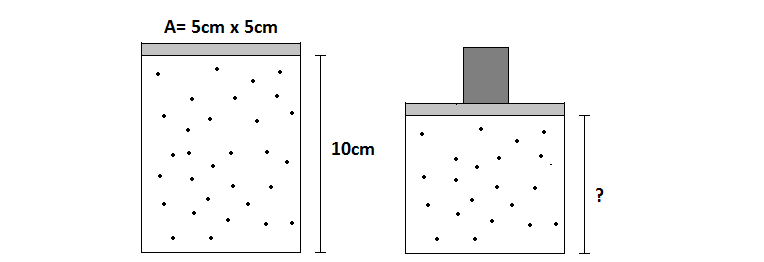
**Example balloon**

Ignoring PE of elastic balloon material, suppose we have an r = 1m balloon, filled with air at room temperature T = 300 and at atmospheric pressure. To what temperature must we heat it up to expand its radius to 1.3m. How much heat will this require?

Let’s get # of mols first. n = PV/nR = (101kPa)(4/3πr3)/nR = 170. Then T = pV/nR = 101kPa∙(4/3)π(1.3)3/nR = 658K.

And then from 1st law: -W + Q = ΔE → -patmΔV + Q = (f/2)nRΔT → -nRΔT + Q = (f/2)nRΔT → Q = (1+f/2)nRΔT = (1+5/2)(173)(8.31)(658-300) = 1.8MJ.

**Question 4**. Suppose you keep a triatomic gas in the well-insulated box with a movable piston at atmospheric pressure. Then you place a 5.1kg mass on top of it. Assuming the gas remains well insulated so that no heat can escape or enter, what will be the new equilibrium height of the gas?



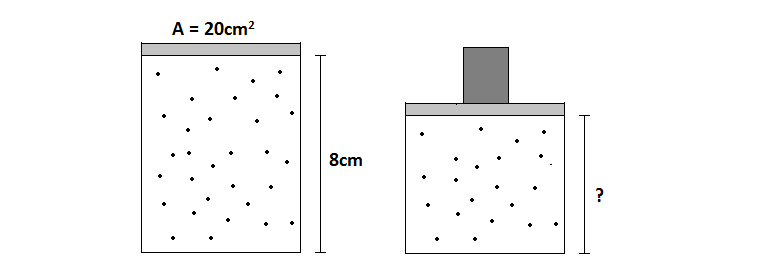
So the pressures must balance,



and the new pressure is related to the old via P1V1γ = P2V2γ → P2 = P1(V1/V2)γ, where γ = 1+2/f. And P1 = Patm.  So,



1. Suppose you keep a diatomic gas in the well-insulated box with a movable piston at atmospheric pressure. Then you place a 2kg mass on top of it. Assuming the gas remains well insulated so that no heat can escape or enter, what will be the new equilibrium height of the gas?



So the pressures must balance,

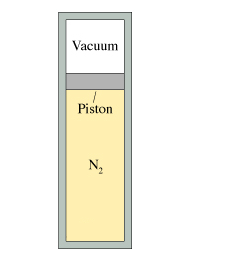


and the new pressure is related to the old via P1V1γ = P2V2γ → P2 = P1(V1/V2)γ, where γ = 1+2/f. And P1 = Patm.  So,



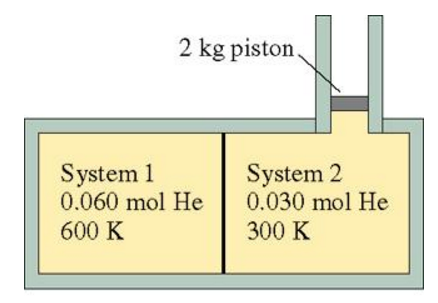
**Question 4**.

Suppose N2 gas is in the following container, supporting the weight of a 2kg piston. If 15J of energy is poured into the gas, how high will the piston rise?





**Question 11**. The figure shows two compartments separated by a thin wall. The left side contains 0.060 mol of helium at an initial temperature of 600K and the right side contains 0.030 mol of helium at an initial temperature of 300K. The compartment on the right is attached to a vertical cylinder. A 10cm diameter, 2.0kg piston can slide without friction up and down the cylinder. The volumes of the compartments are unknown. The exterior pressure is 1.0 atm. How high will the piston rise as the gasses come to thermal equilibrium?



Applying first law to system 1 we have: Q1 – W1 = ΔE1 → -Q – 0 = (3/2)n1RΔT1 (since it will lose heat we define Q1 as -Q). And applying the first law to system 2 we have: Q2 - W2 = ΔE2 → Q – p2ΔV2 = (3/2)n2RΔT2. For the second system we can say p2ΔV2 = n2RΔT2 and so have:

Q – n2RΔT2 = (3/2)n2RΔT2 → Q = (1+3/2)n2RΔT2. Filling the system 1 equation into the system 2 equation we get (3/2)n1RΔT1 = -(1+3/2)n2RΔT2 → -(3/2)(0.060)(8.31)(T-600) = (1+3/2)(0.030)(8.31)(T-300) → T = 464K.

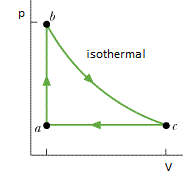
To get h, we can go back to p2ΔV2 = n2RΔT2 x → ΔV2 = n2RΔT2/p2 → h = n2RΔT2/p2A. Filling in n2 = 0.03, ΔT2 = 464-300, p2 = 101250 + (2)(9.8)/A, and A = π(0.052), we get h = 0.05m = 5cm.

**Engines**

**Question 4**. Let 1.2 moles of diatomic gas in a heat engine be taken through the following process. What is the efficiency of the engine?

pa = 150kPa, Va = 0.04m3, Ta = 600K

pb = 300kPa, Vb = 0.04m3, Tb = 1200K



First we need to fill in the c values….so



and then figure out the work done. This is:



The heat input comes from process a → b and b → c. We’ll figure these out individually,



and,



So the total heat added is:



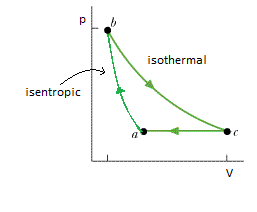
and so the efficiency is:



**Question 4**. Let 1.2 moles of triatomic gas in a heat engine be taken through the following process. What is the efficiency of the engine?

pa = 150kPa, Va = 0.04m3, Ta = 600K

pc = 150kPa, Vc = 0.08m3, Tc = 1200K



Well we need to fill in the b-values:



and then figure out the work done. This is:



The heat input comes from process b → c. We’ll figure these out individually,

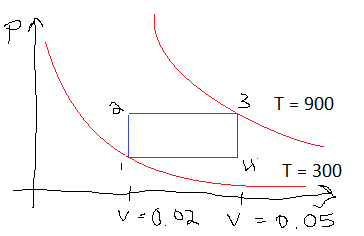


and so the efficiency is:



**Example: Rectangular cycle**

Suppose we have 1.2 mol of gas which can operate between T = 300 and T = 900. Let’s construct a rectangular cycle operating between these temperatures. In construction below, must be careful that V4 and V3 are in between the curves. Basically, you choose n, Tc, Th to fix the curves below. And then draw rectangle. Then V4 and V2 are arbitrary, and can then figure out what the rest must be (i.e. pressures and such).



So have to figure out the points.

1) P1 = nRT1/V1 = (1.2)(8.31)(300)/(0.02) = 150kPa

3) P3 = nRT3/V3 = (1.2)(8.31)(900)/0.05 = 180kPa

2) T2 = P2V2/nR = (180kPa)(0.02)/nR = 360K

4) T4 = P4V4/nR = (150kPa)(0.05)/nR = 750K

**Net work done by gas**

And now we can calculate the net work done by our gas.



Since the weight has a mass m = 1000 kg, this means that we could raise it h = W/mg = 9cm every cycle.

**Heat input**

What is the heat that we have to input? We input heat during processes (2) → (3), and during (1) → (2). So the net heat input is:



The heat input during the first process can be determined. From the 1st law it is:



We can determine the heat input during (1) → (2) using the 1st law of thermodynamics,



Therefore the net heat input is:



**What is change in energy of our gas?**

This is easy to answer. The change in energy of our gas is:



But note that the temperature of our gas doesn’t change from beginning to end of cycle. This is because at beginning and end, it has the same P and V. And since PV = NkT, that means it has the same T as well. So ΔT = 0 → ΔE = 0. This is *always* the case for a cyclic process.

**What is the heat output, QC?**

From the 1st law, over the entire cycale, we have:



**What is the total change in entropy of our system?**

Our system comprises the machine and the heat / cold sources that transfer heat to and from the machine. This is a closed system since energy is transferred only between these objects. And now we’d like to know the net change in entropy of this system. Well,



**What is the efficiency of our machine?**

The efficiency of our engine is defined as the net work done divided by the heat input.

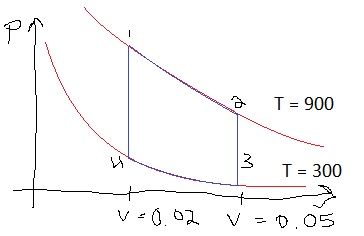


and so this works out to be:



**Method 2: Isothermal cycle**

The basic idea of the isothermal cycle is to raise and lower the piston at a constant temperature rather than at a constant pressure. On the P-V diagram it would look like this (in green), overlayed against the isobaric cycle for comparison’s sake.



Assuming we have the same number of moles of particles in our diatomic gas again, n = 1.2 mol… note that once choose n, Tc, Th, then curves are set. Then V4 and V2 are arbitrary once again.

4) P4 = nRT4/V4 = nR∙300/0.02 = 150kPa

3) P3 = nRT3/V3 = nR∙300/0.05 = 60kPa

2) P2 = nRT2/V2 = nR∙900/0.05 = 180kPa

1) P1 = nRT1/V1 = nR∙900/0.02 = 450kPa

**Net work done by gas**

The net work is the sum of the works done during each of the 4 processes.



So we do less work in this cycle than in the previous one. Now let’s see how much is input.

**Heat input**

What is the heat that we have to input? We input heat during process (1) → (2), and during (4) → (1). So the net heat input is:



We can determine the heat input during (1) → (2) using the 1st law of thermodynamics,



And now let’s determine the heat input for process (4) → (1). This is:



So, the net heat input is:



Observe that we have to input a lot less heat using this cycle than using the previous one.

**What is change in energy of our gas?**

This is easy to answer. The change in energy of our gas is:



But note that the temperature of our gas doesn’t change from beginning to end of cycle. This is because at beginning and end, it has the same P and V. And since PV = NkT, that means it has the same T as well. So ΔT = 0 → ΔE = 0. This is *always* the case for a cyclic process.

**What is the heat output?**

From the 1st law, we have,



Let’s apply this to the entire cycle. Then we must have,



**What is the change in entropy of our system?**

Our system comprises the machine and the heat / cold sources that transfer heat to and from the machine. This is a closed system since energy is transferred only between these objects. And now we’d like to know the net change in entropy of this system. Well,



The reasoning is this. First, the net change in entropy of our gas is 0 because it goes through a cyclic process. It begins and ends in the same state (T1, V1) and so it must have the same entropy at the end as it does in the beginning. The ice bath increases in entropy because it accepts an amount of heat Qout = 14 kJ at a temperature of 300K (we make the standard simplifying assumption that the ice bath is so large that accepting that heat doesn’t change its temperature appreciably). The fire decreases entropy because it gives an amount of heat Qin = 14 kJ away at temperature T = 600K.

**What is the efficiency of our engine?**

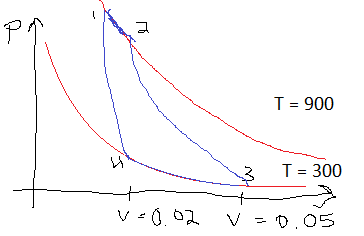
The efficiency of our engine is



So this is quite a bit more efficient than our previous cycle.

**Method 3: Carnot cycle**

The Carnot cycle is shown below, in red, overlaid against the previous two cycles. Once you choose n, Tc, Th then isothermal curves are set. And you can choose V4, V3 as before. But note that even though it looks like V2 > V4 in the diagram, it actually isn’t.



So

4) P4 = nRT4/V4 = nR∙300/0.02 = 150kPa

3) P3 = nRT3/V3 = nR∙300∙0.05 = 60kPa

1) V4T4f/2 = V1T1f/2→ V1 = 0.02(300/900)5/2 = 0.00128m3

P1 = nRT1/V1 = nR∙900/0.00128 = 7.03 MPa

2) V2T2f/2 = V3T3f/2 → V2 = V3(T3/T2)f/2 = 0.05/35/2 = 0.00321m3

P2 = nRT2/V2 = nR∙900/0.00321 = 2.80 MPa

**Work done by gas**

The net work is the sum of the works done during each of the 4 processes.



We have:



So we do even less work in this cycle than in the previous one. Now let’s see how much heat is input.

**Heat input**

Heat is input during the isothermal expansion only (1) → (2). From the 1st law, we have,



**What is change in energy of our gas?**

This is easy to answer. The change in energy of our gas is:



But note that the temperature of our gas doesn’t change from beginning to end of cycle. This is because at beginning and end, it has the same P and V. And since PV = NkT, that means it has the same T as well. So ΔT = 0 → ΔE = 0. This is *always* the case for a cyclic process.

**What is the heat output?**

From the 1st law, we have,



Let’s apply this to the entire cycle. Then we must have,



**What is the change in entropy of our system?**

Our system comprises the machine and the heat / cold sources that transfer heat to and from the machine. This is a closed system since energy is transferred only between these objects. And now we’d like to know the net change in entropy of this system. Well,



**What is the efficiency of our engine?**

The efficiency of our engine is

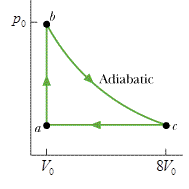


6. The figure shows a cycle through which 1.20 mole of a monatomic ideal gas is taken. Process *bc* is an adiabatic expansion. And *p0* = 12.0 atm, *V0* = 5.0 x 10-3 m3. Find the efficiency of the cycle. Determine p, V, and T at points a, b, c.

pa = 37.8 kPa Va = 5×10-3 m3 Ta = 19K

pb = 1.21 MPa Vb = 5×10-3 m3 Tb = 607K

pc = 37.8 kPa Vc = 4×10-2 m3 Tc = 152K



We need to find the pressures/volumes, temperatures.



and,



and finally,

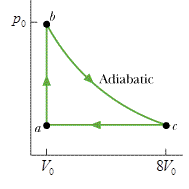


7. Consider the process in the figure again. Let there be 0.7 moles of monatomic gas this time. Then determine the efficiency of the cylcle, given:

pa = 15.8kPa, Va = 0.002m3, Ta = 5.43K

pb = 505kPa, Vb = 0.002m3, Tb = 1740K

pc = 15.8kPa, Vc = 0.016m3, Tc = 43.4K



Now we have to find the work done by the cycle, and heat intake. We'll break it down into steps. Note that I’m designating W as the work done by the gas, and so the first law would read Q – W = ΔE.



And,



and last,



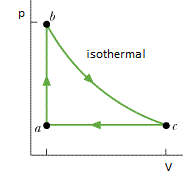
So the efficiency is:



**Question 4**. Let 1.2 moles of diatomic gas in a heat engine be taken through the following process. What is the efficiency of the engine?

pa = 150kPa, Va = 0.04m3, Ta = 600K

pb = 300kPa, Vb = 0.04m3, Tb = 1200K



First we need to fill in the c values….so



and then figure out the work done. This is:



The heat input comes from process a → b and b → c. We’ll figure these out individually,



and,



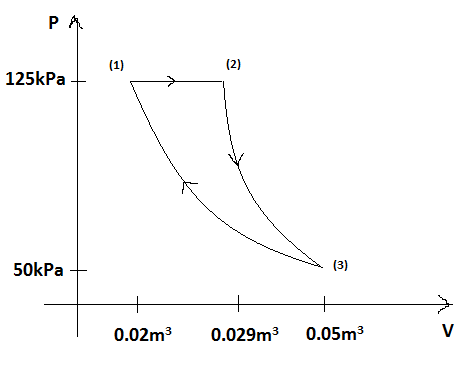
So the total heat added is:



and so the efficiency is:



**Question 5**. Let an n = 0.7 mol monatomic gas be takeng through the following processes: (1) → (2) be isobaric, (2) → (3) be isentropic, and (3) → (1) be isothermal, at the pressures and volumes shown below. What is the efficiency of this engine?



First we need to get the temperatures,



Then the work, heat done for each process is:



and,



and last,

ne mo



So the efficiency is:



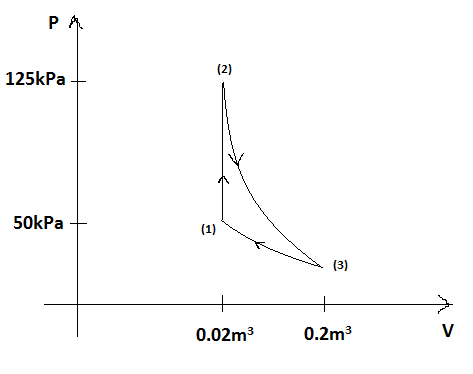
**Question 6**. What would be the efficiency of a Carnot engine operating between the same high and low temperatures as the cycle above? How much heat would have to be input into such an engine to raise a 200kg weight 150m?



and,



4. Let an n = 0.7 mol diatomic gas be takeng through the following processes: (1) → (2) be isometric, (2) → (3) be isentropic, and (3) → (1) be isothermal, at the pressures and volumes shown below. What is the efficiency of this engine?



First we need to get temperatures,



Calculating work,



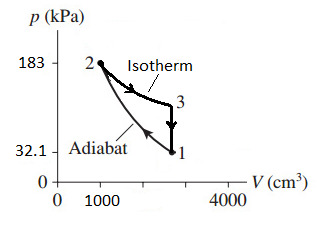
and the heats



and so efficiency,



**Question 2**. Consider an engine using 0.30 mol of a monatomic gas as the working fluid. The engine cycle is shown below. V1 = 2840cm3. Calculate the efficiency of this cycle.



We’ll need the temperature at point 2. This is



And so then the efficiency is η = W/Qin. So let’s get W first



Heat will be *added* during the isothermal process alone. And from the first law, this heat will be the same as the work. So Qin = 191J. So



**Question 6**. What would be the efficiency of a Carnot engine operating between the same high and low temperatures as the cycle above? How much heat would have to be input into such an engine to raise a 200kg weight 150m?

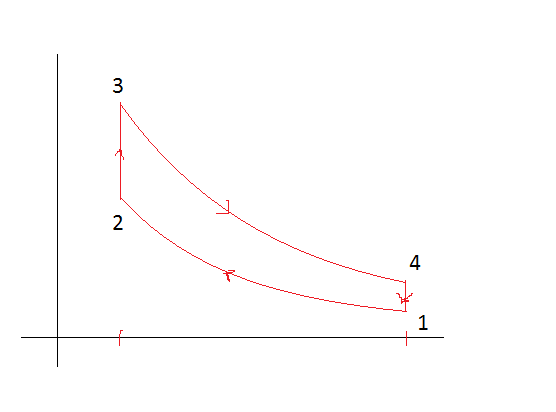


and,



**Example**

Consider barrel of cannon that is 1.5m long with cross section radius r = 5cm, that you use to fire projectile with mass m = 5kg. Process is this (1) stuff shot down into the barrel assumed to be airtight, with maximum force F = 800N – assume isothermal process (2) ignite powder to deliver some amount of heat Q = 30kJ – assume isometric process b/c very quick or have a latch (3) rapid expansion of gas to push shot out of barrel – assume isentropic (4) expel gas and draw in new (or simply cool gas) back to step one – assume isometric again. Questions are: (a) how far down the barrel can you push the shot? (b) fill out process in pV diagram – labelling p, V, T, E, S; as well as all W’s and Q’s (d) get net work and net Q as well (e) what velocity will the cannonball have?



P1 = 101kPa, V1 = L∙πr2 = 0.012, T1 = 300K; n = PV/RT = 0.477; E1 = 3kJ; S1 = 39J/K.

P2 = Patm + 800/πr2 = 203kPa, T2 = 300K, V2 = nRT2/P2 = 0.00587 → ℓ = V2/πr2 = 0.75m; E2 = 3kJ; S2 = 36J/K

V3 = 0.0078m3; ΔE = Q → ΔT = Q/(f/2∙nR) → T3 = 3330K; p3 = nRT3/V3 = 2.25MPa; E3 = 33kJ; S3 = 60J/K477

V4 = 0.012m3; p4 = p3(V3/V4)γ = 847kPa; T4 = 2520K; E4 = 25kJ; S4 = 60J/K

And then

W12 = nRTln(V2/V1) = -830J; Q12 = -830J

W23 = 0; Q23 = 30kJ

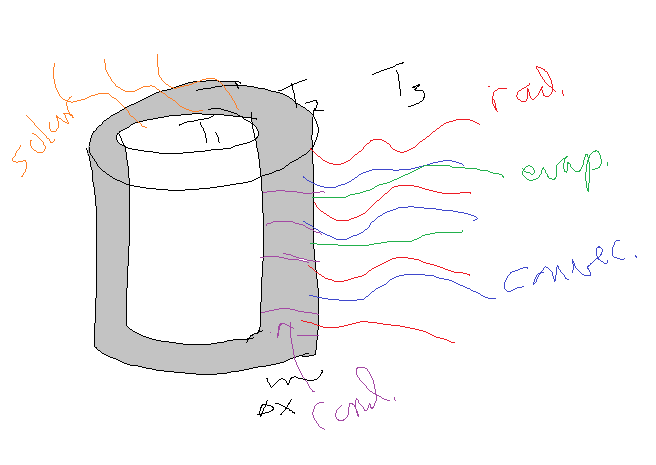
W34 = Δ(PV)/(1-γ) = 8kJ; Q34 = 0

W41 = 0; Q41 = ΔE = -22kJ

Speed = ? Well Wgas - Watm = ΔKE → 8kJ – 101kPa∙(0.012-0.0078) = ½mv2 → 7.58kJ = (1/2)(5)v2 → 55m/s.

Efficiency of process is η = Wnet/QH = (8kJ – 0.83kJ)/30kJ = 24%.

**Humans**

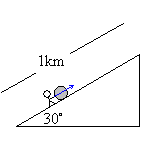


If in equilibrium, then should have:



and so really should only consider the last 4 perhaps, or first one, depending on where you draw system boundaries, but not all 5. T2 would be skin temperature of course, or jacket temperature, etc. And T1 would be core temperature.

15. Suppose you’re Sissyphus, the king in Greek mythology condemned to role a stone up a hill for eternity. Your mass is m = 60kg, and you role a 50kg stone up a 30˚ incline through a distance of 1km. Assuming η = 20%, and ignoring internal work, how much internal chemical potential energy do you burn (in kcal) to accomplish this task?



The work you do can be calculated by the work-formula.



which represents only 20% of your energy expenditure, which is:



16. Suppose you’re still Sissyphus. Given that the specific heat capacity of the human body is 3.47kJ/kg, and ignoring internal work again, what would be your temperature increase from doing all that work if you couldn’t transfer any heat?

The other 80% of the internal energy spent must be heat since, from the 1st law of thermodynamics we have:



17. Model yourself as a cylinder with height h = 1.7m, and radius 12cm. Ignoring internal work once again, and considering heat transfer via convection alone, how much would your temperature go up now? You may presume your skin temperature doesn’t change, that the ambient temperature is 30C, and that it takes you 20 minutes to roll the stone up the hill).



**Problem**. Now suppose that the major form of heat transfer is radiation. Presuming skin temperature is 35C and ambient temperature 30C, as before, and letting ε = 1, and Δt = 20s. How would be your change in temperature now? Ignore internal work.



**Problem**. Now suppose that you eliminate heat via evaporation. How much water would you sweat to keep your temperature from changing during this process? Ignore internal work.



**Question 1**. Suppose you use a stair climber, and climb the equivalent of 850m. And further suppose, for the sake of discussion, that your mass is about 70kg, your efficiency about 22%, and your heat capacity about 3.5 kJ/kg∙K. If your core temperature increases about 3 degrees, and you lose heat through evaporation of sweat, how much water would you lose (in kg). Ignore internal work.



**Example**

Running burns about 100 kcal per mile, meaning that ΔPEchem. = 100kcal per mile, approximately. Assuming you run 6 miles, how much water would you sweat?

We’ll use the 1st law again. And we’ll take you as the system. During the 6 mile run you will lose about 600 Calories of energy. This will show up in a certain amount of work Wnc. done and an amount of heat, Q, given off. .



11. Suppose you work out on a stair climber. You climb one stair (height of stair = 0.30m) every second. If your mass is 70kg, (a) how much work will you do in 30 minutes? and (b) how many kcalories of internal energy will you ‘burn’ in those 30 minutes?

The amount of work you do in 30 minutes would be:



and we have:



**Example**

Suppose you (m = 65kg) jump up into the air 0.5m once every 2s. How long would it take to burn 500 Cal?

To get ΔEint. we need to calculate the work you do. The work is this. Every jump requires an amount of work equal to W = mgh = (65)(9.8)(0.5) = 318 J. You make a jump every 2s, and so the total amount of work you do in a given time *t* is: W = (318)(t/2) J = 159t J. The work is equal to 0.2ΔEint.. ΔEint. will be equal to 500Cal when,



3. Suppose you do internal work at rate W­­int = 15J/s, and you can be modelled as a cylinder of height h = 1.7m, and radius R = 12cm, with skin temperature T = 35C. (a) At what ambient temperature would you feel most comfortable? In other words, at what ambient temperature would you radiate heat at just the right pace to keep your temperature constant? Assume radiation is the only source of heat loss and takes place via all surfaces, and that ε = 1. (b) Assume now that you exercise on a bike, exerting a relatively constant power output of 200W. Including thermal radiation and evaporation, how much water will be lost in one hour.



(b)



**Question 1**. Suppose you use a stair climber, and climb the equivalent of 850m. And further suppose, for the sake of discussion, that your mass is about 70kg, your efficiency about 22%, and your heat capacity about 3.5 kJ/kg∙K. If your core temperature increases about 3 degrees, and you lose heat through evaporation of sweat, how much water would you lose (in kg).



**Question 1**. Suppose you use a stair climber, and climb the equivalent of 850m. And further suppose, for the sake of discussion, that your mass is about 70kg, your efficiency about 20%, and your heat capacity about 3.5 kJ/kg∙K. How many calories do you burn? Ignore internal work. If you wore a thick jacket to eliminate evaporative cooling and to minimize thermal radiation and conduction as means of cooling off how much would your core temperature have increased? If your core temperature increases about 3 degrees, and you lose heat through evaporation of sweat, how much water would you lose (in kg).

calories burned is just



If you eliminate all heat transfer then we’ll have:



And then if we assume heat transfer via evaporation,



4. Suppose you’re outside in the cold (T = -20C). How thick a jacket will you need to stay warm (meaning your temperature doesn’t change) if its material has thermal conductivity k = 0.04 W/m∙K? You can take your temperature to be 35C. And you can suppose that the area of the jacket is A = 0.7m2. Just consider heat transfer via conduction.



5. Suppose you’re running at an 8min/mile pace. If we approximate you as cylinder with height 1.7m, and radius 12cm, for what ambient temperature would radiation be sufficient to keep you from heating up? Take your skin temperature to be 35C. Take ε = 1. Don’t ignore internal work.



1. Of the approximately 2000 Calories you eat every day, about 80% ends up as internal thermal energy. This energy would raise your temperature to dangerous levels and so must be gotten rid of. The main way your body rids itself of this energy is via radiative heat transfer. So pretend you’re a cylinder 1.75m tall, and 75cm in circumference. If all surfaces of you (the cylinder) radiate heat, then at what ambient temperature would you radiate all of this heat away in a day? You can take your emissivity to be 1, and your skin temperature to be 34°C. Note this is the temperature at which you would feel most ‘comfortable’: at hotter temperatures you wouldn’t radiate heat away quickly enough and would get hot, and at colder temperatures, you would radiate heat too quickly and get cold.

So first your radius is: R = C/2π = 75/2π = 12cm. And then,



10. A person burns about 2000 Calories per day. As was mentioned in class, about 4× as much heat as work is generated from this internal energy. If the heat generated is not dissipated, then the temperature of the body will increase, resulting in bad stuff. Luckily for you, a lot of this heat is radiated away since the temperature of you Tyou = 35°C (at the surface perhaps) is higher than the surrounding temperature, Toutside. So the question is this, if the human body has a heat capacity of 3.47kJ/kg, what temperature Toutside would result in the heat being radiated away at the same rate as it is generated? This ought to be the temperature at which you feel most ‘comfortable’, naked of course. Take your surface area to be 1.7m2, and your emissivity to be 1. And one Calorie is 4186 J.

The rate at which you generate heat is:



and this is to match the rate at which you radiate heat. This is:



which would be, in Celsius, T = 301 – 273 = 28°C ≈ 82°F.

Could write from 1st law as:



**Question 12**. Consider your surface area to be 1.4m2 and that you have a skin temperature T = 35C.  You are riding a bike, exerting a constant power output of 180W.  Including internal work, thermal radiative heat transfer, convective heat transfer, and evaporative heat transfer, how much water will be lost in one hour?  You can take the ambient temperature to be 301K, εTH = 1, and the convection coefficient to be h = 20 W/m2K.



**Question 6**.

You pedal on a bike, exerting a power of 180W. Ignoring internal work, assuming a metabolic efficiency of 20% and that evaporation is the primary heat loss mechanism, how many kcal will you burn in 45min, and how much water will you sweat?

Calories burned can be calculated via:

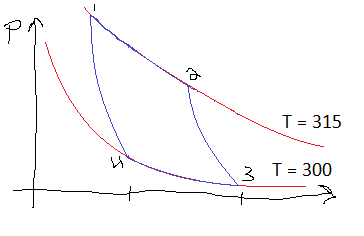


and to get the water lost, we can apply the first law,



**Refrigerators**

Suppose you’re trying to keep your room cool at T = 300, vs. Toutside = 315. If the refrigerator you use operates on a Carnot cycle, then what is its coefficient of performance?



Well that’s:



Could ask what power does the engine operating on the refrigerator have to exert to keep room cool if the insulation between the walls is k = whatever, and cross section area is A = whatever, and thickness of walls is Δx = whatever?

Well,



And we know that Qout/Wnet = κ → Wnet = Qout/κ → dWnet/dt = dQout/dt ∙ 1/κ. So we have:



**Question 7.**

A Carnot refrigerator operates between reservoirs at 50C and -15C. If it uses a power of 15 Watts, how long will it take to cool 400mL of soda from 20C to 5C? You make take the soda to have the thermodynamic properties of water for these purposes (and note 1mL = 1g)

First we need to figure out the rate at which the refrigerator extracts heat from its interior. We have:



and also,



and now applying the first law to the soda,



**Question 4**. A Carnot AC unit operates between reservoirs at 35°C and 20°C and is used to cool a room. For simplicty, assume the room is a 4m×4m×4m cube and that it conducts heat through all 6 sides, which are 3cm thick and have a thermal conductivity coefficient of 0.05 W/m°K. Let the interior temperature of the room be T = 20°C, and the outside temperature be T = 35°C. What power must the AC unit operate at to keep the temperature of the room constant?

First we need to figure out the rate at which the refrigerator extracts heat from its interior. We have:



and also,



And the rate at which heat is added to the room via conduction is:

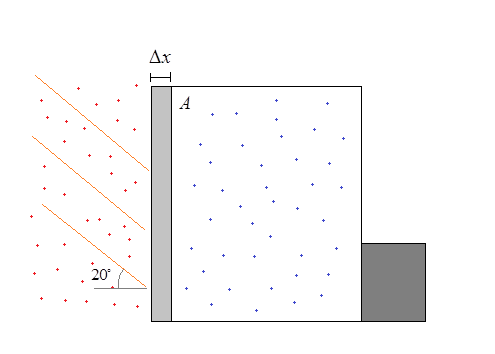


and now applying the first law to the room,



**Refrigerator problem**

Don’t remember the original #’s, so I’ll just make up new ones. Suppose we have a sunroom, which lets in heat and sunlight through a large window. We’ll say the window width is Δx = 2cm, its area A = 12m2, and its thermal conductivity is k = 0.10 W/m°K. We’ll say the outside air is at temperature 40°C, and the air inside the room is at 25°C. And we’ll say that the solar radiation (orange lines) has intensity I = 1200 W/m2, makes and angle of 20° with respect to the window’s normal line, and that 15% of the radiation is reflected off the window. So question is, if we have a (reverse) Carnot AC unit pumping heat out of the room, what power must it exert to keep the room’s temperature constant at 25°C?



And set up would look like this. Applying first law to the air in the room, we have:



where we note that ΔEair = 0 because the temperature of the room is constant. OK, now QAC is related to the AC unit’s coefficient of performance via:



Moreover, since the AC unit is a Carnot unit, the coefficient of performance will be equal to,



and so,



So filling this in, along with the formulas for conduction and solar radiation we have:



and this the power, since power = W/Δt, i.e. dW/dt.